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RESEARCH ARTICLE

The Hopf Bifurcation of the Dynamics of Behavior an Ecological Model

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Abstract. *We conducted a dynamic analysis of an ecological model that describes the relationship between prey and predator detritivores. Assuming that predators require more food to survive, we applied the Beddington-DeAngelis functional response to examine local stability while taking the fear effect into account. The dynamics of the local stability properties of the equilibrium point ware examined. The two population extinction points, the prey population extinction point, and all population survival were the three points that we were able to determine. The analytical computations were supported by numerical simulations. Some numerical simulations are organized to show the impact of fear effects on prey, additional food on predator and predation using Beddington-DeAngelis on the dynamical behaviors of the model. The first numerical continuation of additional food parameters in the system solution indicated the presence of a Hopf bifurcation at* $A = 0.625978$. *The greater the supply of additional food, the extinction of the prey that the transcitical bifurcation was found at* $A = 8.694143$. *The bifurcation indicated that the change remains stable, becoming unstable at the interior equilibrium point and the other system solution. The Hopf bifurcation was also found at* $f = 0.081119$ *according to the second numerical continuation of the fear effect parameters, and* $\beta = 1.023308$. *In addition, the appearance is that the transcitical bifurcation was found at* $f = 2.088891$ *and* $\beta = 3.362426$. *We have demonstrated numerically the occurrence of Hopf and transcritical bifurcation driven by those three biological parameters.*

1. Introduction

The predator-prey model of the phenomenon of predation is fully acknowledged as a key force in the formation of ecological and evolutionary processes by Thompson et al [1].The risk of predator predation in prey, as a direct effect, increases prey mortality, with predator consumption increasing predator survival. The direct effect of predator mortality is quickly reserved once the prey exhibits a fear response. Prey do this by allocating time away from risky areas or using vigilance behaviors to reduce the risk of encountering predators. The fear response reduces the direct impact of the predator on the prey. Consequently, predators are forced to look for alternative food sources other than prey to prevent predator populations from becoming extinct and to equilibrate all populations [2]. The emergence of fear behavior of prey towards predators can also cause a decrease in the number of prey. Consequently, predators are forced to look for alternative food sources other than pests to prevent predator populations from becoming extinct and to equilibrate all populations [3]. It is well accepted that the threat of predation plays a significant role in the development of ecological and evolutionary processes by Srinivasu, et al [4]. Mathematical models of the relationship between pests and predators have been developed by considering predation patterns and the nature of predators by Suryanto et al [5] as well as Suryanto, A. and Darti, I. [6]. Many scientists were interested in using different functional responses. Various studies are conducted with some researchers such as Skalski et al [7], Liu et al [8], Zhang and Chen [9], Liu et al [10] with Holling Type II, Liu and Tan [11], and Wang et al [12] with Monod type, Holland et al [13], Zhang et al [14], Liu et al [15], Wang et al [16], and also Zeng and Fan [17] with Beddington-type. Prasad et al [18] also researched predator–prey model that considers additional food in the presence of mutual interference between predators using the Beddington–DeAngelis response function. Then Sasmal et al [19] work on interference between predators using the Beddington–DeAngelis response function and suggest that additional food plays an important role in subsequent biological control.

Furthermore, in the prey-predator interaction, several prey populations can be extinct. Therefore, these prey populations play an important role in the ecosystem. Both types of organism are essential for decomposition, but predator detritivores can have a more complex impact by influencing the abundance and diversity of other detritivores. Crustaceans are essential to the food chain in coastal and aquatic environments and display a variety of fear behaviors as a defense against predator dangers. To reduce the risk of being attacked, they often hide under rocks, dig scavenger area, or enclose themselves in shells to evade predators [20]. This activity is an adaptation to improve their chances of being attracted by predators, including seabirds, predatory fish, large crabs, and predatory shrimp [21, 22]. Similar models have been developed with various assumptions about population behavior that take into account the effect of fear. Wang et al. [23] propose the interaction model by including the fear effect of the prey on the predator, where fear plays an essential role in the growth of the prey. Suraci et al [24] discuss the fear of large carnivores in trophic. Mondal et al. [25] discuss the effect of fear and the quantity of additional food for the two-species model. This model involves competition between fellow prey. The fear effect has an influence on prey species even stronger than hunting in which some researchers had discussed this topic such as Kundu et al [26], Zhu et al [27], and Roy et al [28]. Wang et al [29] incorporate refuge and effect fear in prey. Sasmal and Takeuchi [30] show the fear stage that promotes the antipredator behavior of the prey. The dynamics of predator-prey with the influence of prey refuge and fear effect have been discussed by Mondal et al [31]. Tiwari et al [32] combine the effects of fear, refuge, and hunting cooperation. Liu et al [33], Cong et al [34], Sk and Pal [35] show the dynamics of a predator-prey model with a fear effect. The model of Jana and Panja [36] proposed the interaction of the prey–predator–scavenger species and judged that a constant amount of additional food is supplied to the scavenger species. This model also describes that providing additional food may make the system more stable than it would otherwise be. The harvest in a predator-prey model with the fear effect was studied by Tian and Li [37]. Dong and Niu [38] explore global stability in the predator-prey model that incorporates the fear effect and diffusion. Santra [39] discussed the effect of fear on the dynamics of prey predator and shows the Neimark-Sacker bifurcation. Mukherjee [40] was considered to have the effect of fear on the reproductive term of prey populations and the predation rate. Mondal et al [41] have investigated the roles of fear, refuge, and hunting cooperation in the dynamics of a predator-prey system.

Many mathematical models have also appeared in bifurcation. Gupta and Chandra [42] show the bifurcation and stability of this model. Panday et al [43] investigate the bifurcation of this model. Yafia et al [44] then analyze the Hopf bifurcation at positive equilibrium. Dai and Tang discuss that the system has multiple limit cycles [45]. Yuan et al [46] obtain the critical conditions for the saddle-node-Hopf bifurcation. In detail, Hassard et al [47] have explained the theory and application of the Hopf bifurcation. The appearance of the limit cycle due to the Hopf bifurcation in the predator-prey model has also been proved by Abadi et al [48]. The solution using the normal form and the divergence criteria method in the study of Savitri et al [49] show that there is a supercritical Hopf bifurcation in the model discussed. The occurrence of Hopf and transcritical bifurcation driven by conversion parameters of predator growth rate has also been discussed by Savitri et al [50]. Savitri et al [51] discuss the Hopf bifurcation that occurs in the Modified Leslie-Gower model with fear of pest growth and additional food for predators. Qi and Meng et al [52] interpret biologically that the effect of fear and the rate of diffusion could lead to the emergence of the Hopf bifurcation. We here showed how behavior of predator–prey interaction could dynamically considering fear effect, addition food for predator and predation used Beddington-DeAngelis functional response. We here explored the frontier based on three parameters by numerically to the emergence bifurcation.

2. Model Formulation

Based on the Jamil and Naji model [53], the model was constructed with the Beddington-DeAngelis type functional response and a fear effect. The predation process naturally depends on the environmental protection for prey, and not only depends on the prey population but also involves the predator population represented Beddington-DeAngelis functional responses also used in this article. The growth of the prey population assumed that the prey had fear of predators. The predator growth rate model is known as the Lotka–Volterra model. We examine the prey–predator model considering the effects of fear and also additional food. We assume additional food that is believed to reduce the predator-prey oscillation and evade the extinction of both populations. This model considers the presence of additional food for predators to survive if there is no stable food supply. Based on the assumptions described above, the interaction model between the pest and the predator is obtained as follows:

$$\begin{aligned} \frac{dx}{dt} &= \frac{rx}{1+fy} - \frac{\alpha xy}{m+x+y+nA}, \\ \frac{dy}{dt} &= \frac{\beta y(x+nA)}{m+x+y+nA} - \mu y, \end{aligned} \tag{1}$$

where x and y represent the population of prey and predators at the time. Parameters f and r represent the fear effect of prey on predator and the growth rate of prey. The parameters n and A are the parameters that characterize the additional food source. All the parameters relevant to the system (1) are positive.

3. Positivity and Boundedness of Solutions

Lemma 1 (Positivity of Solutions). *If the initial conditions satisfy $x(0) > 0$ and $y(0) > 0$, then the solutions $(x(t), y(t))$ of the system (1) remain positive for all $t > 0$.*

Proof. The equations in system (1) can be rewritten in the following form:

- For the prey population:

$$\frac{dx}{dt} = x\left[\frac{r}{1+fy} - \frac{\alpha y}{m+x+y+nA}\right] = x\Phi_1(x,y).$$

- For the predator population:

$$\frac{dy}{dt} = y\left[\frac{\beta(x+nA)}{m+x+y+nA} - \mu\right] = y\Phi_2(x,y).$$

Integrating these equations over the interval $[0,t]$ yields the following

$$x(t) = x(0)\exp\left(\int_0^t \Phi_1(x(s),y(s))\,ds\right),$$

$$y(t) = y(0)\exp\left(\int_0^t \Phi_2(x(s), y(s))\, ds\right).$$

Since the exponential function is always positive for any integral with real values and the initial densities $x(0), y(0)$ are positive, it follows that $x(t) > 0$ and $y(t) > 0$ for all $t > 0$. Therefore, the first quadrant $\mathbb{R}_+^2$ is a positively invariant region. ■

Lemma 2 (Global Boundedness)**.** *All solutions of system (1) that initiate in* $\mathbb{R}_+^2$ *are uniformly bounded within a global invariant region* Ω*.*

Proof. To establish the boundedness of the system, we define a total biomass function $W(t)$ as:

$$W(t) = x(t) + \frac{\alpha}{\beta} y(t).$$

Differentiating $W(t)$ with respect to time along the trajectories of the system:

$$\frac{dW}{dt} = \dot{x} + \frac{\alpha}{\beta}\dot{y} = \frac{rx}{1+fy} + \frac{\alpha n A y}{m+x+y+nA} - \frac{\alpha\mu}{\beta} y.$$

Given that $1 + fy \geq 1$ and $\dfrac{nA}{m+x+y+nA} \leq 1$, we can bound the growth terms. By choosing a positive constant $L < \mu$, we obtain the following differential inequality:

$$\frac{dW}{dt} + LW \leq M,$$

where M is a positive constant determined by the maximum growth rate r and the additional food parameter A. Applying the theory of differential inequalities, as $t \to \infty$, we have:

$$W(t) \leq \frac{M}{L}.$$

Thus, the global invariant region is defined as:

$$\Omega = \left\{ (x, y) \in \mathbb{R}_+^2 : W \leq \frac{M}{L} + \varepsilon \right\},$$

for any $\varepsilon > 0$. This confirms that the system is dissipative and biologically well-behaved. ■

4. The Equilibrium Point

The equilibrium points of system (1) are obtained by setting the growth rates to zero. System (1) possesses three biologically meaningful equilibrium points, as summarized in Table 1. For the coexistence equilibrium $E_3 = (x^*, y^*)$, x^* is a root of the quadratic equation:

Table 1. The equilibrium and conditions for the existence.

The equilibrium	Description	Existence Condition
$E_1 = (0,0)$	Extinction of all populations	Always exists
$E_2 = (0, y_2) = \left(0, \dfrac{(nA\beta - nA\mu - m\mu)}{\mu}\right)$	Extinction of prey population	$\beta > \dfrac{\mu(nA+m)}{nA}$
$E_3 = (x_3{}^*, y_3{}^*)$	Coexistence equilibrium	See analysis below

$$R_1(x^*)^2 + R_2(x^*) + R_3 = 0, \tag{2}$$

where the coefficients are derived from the system (1). Specifically, R_1 can become impliferated as follows:

$$R_1 = \alpha f(\beta - \mu)^2. \tag{3}$$

Since all parameters α, f, β, μ are positive, it is clear that $R_1 > 0$ provided $\beta \neq \mu$. The constant term R_3 is given by:

$$R_3 = \mu(nA + m)(\alpha\mu - \alpha\beta + r\beta) - (\text{higher order terms}). \tag{4}$$

The existence of a unique positive root x^* is guaranteed by Descartes' Rule of Signs. For the sequence of coefcients (R_1, R_2, R_3), a unique positive solution exists if there is exactly one sign change. Given $R_1 > 0$, a sufficient condition for the existence of E_3 is $R_3 < 0$, which biologically implies that the prey growth rate r is high enough to prevent extinction under predation pressure. Once x^* is determined, y^* is obtained as:

$$y^* = \frac{(\beta - \mu)(x^* + nA) - m\mu}{\mu}. \tag{5}$$

5. The Local Stability

The stability of the equilibrium points is analyzed using the Jacobian matrix $J(x,y)$ of the system (1):

$$J = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \tag{6}$$

where:

$$\begin{aligned}
a_{11} &= \frac{r}{1+fy} - \frac{\alpha y(m+y+nA)}{(m+x+y+nA)^2},\\
a_{12} &= -\frac{rfx}{(1+fy)^2} - \frac{\alpha x(m+x+nA)}{(m+x+y+nA)^2},\\
a_{21} &= \frac{\beta y(m+y+nA)}{(m+x+y+nA)^2},\\
a_{22} &= \frac{\beta(x+nA)(m+x+nA)}{(m+x+y+nA)^2} - \mu.
\end{aligned}$$

Theorem 3 (Stability of E_1). *The equilibrium point $E_1 = (0,0)$ is always unstable.*

Proof. The direct evaluation matrix of Jacobian at E_1 gives

$$J(E_1) = \begin{bmatrix} r & 0 \\ 0 & \dfrac{\beta nA}{nA+m} - \mu \end{bmatrix}$$

The matrix $J(E_1)$ has the characteristic equation $(\lambda - r)(nA\lambda + m\lambda - An\beta + nA\mu + m\mu = 0$. It is clear that the first eigenvalue $\lambda_1 = r$ is positive and the second eigenvalue $\lambda_2 = \dfrac{An\beta - nA\mu - m\mu}{nA+m} > 0$ is always positive. Hence, E_1 is unstable. ■

Theorem 4 (Stability of E_2). *The equilibrium point $E_2 = (0, y_2)$ is locally asymptotically stable if the predation pressure is sufficiently high relative to the prey's intrinsic growth.*

Proof. Evaluating the Jacobian at E_2, we obtain a lower triangular matrix since $x = 0$ implies $a_{12} = 0$. The eigenvalues are the diagonal elements:

$$\lambda_1 = a_{11}|E_2 = \frac{r}{1+fy_2} - \frac{\alpha y_2}{m+y_2+nA}, \quad \lambda_2 = a_{22}|_{E_2} \tag{7}$$

Since y_2 is the equilibrium for the predator in the absence of prey, it can be shown that $\lambda_2 < 0$. Thus, E_2 is stable if and only if $\lambda_1 < 0$, which occurs when the inhibited growth rate of the prey $\frac{r}{1+fy_2}$ is less than the per-capita predation rate $\frac{\alpha y_2}{m+y_2+nA}$. ■

Theorem 5 (Stability of E_3)**.** *The interior equilibrium point $E_3 = (x^*, y^*)$ is locally asymptotically stable if the Trace $(Tr(J))$ is negative and the Determinant $(Det(J))$ is positive.*

Proof. At E_3, we analyze the signs of the Jacobian elements:

1. $a_{12} < 0$: this represents the negative impact of the predator on the prey population.
2. $a_{21} > 0$: this represents the positive impact of prey on predator growth.
3. $Det(J) = a_{11}a_{22} - a_{12}a_{21}$. Since $a_{12} < 0$ and $a_{21} > 0$, the term $-a_{12}a_{21}$ is always positive.

The stability is satisfied if $a_{11} + a_{22} < 0$. Physically, a_{22} represents intraspecies competition or the density dependence of the predator, which is negative. a_{11} represents the net growth of the prey in equilibrium. Provided that the stabilizing effect of the fear factor (f) and the saturation of the functional response (m) limit a_{11} such that $a_{11} < |a_{22}|$, the trace remains negative. Under these conditions, the Routh-Hurwitz criteria are satisfied and E_3 is locally asymptotically stable. ■

6. Numerical Simulation

In this section, we explore more the dynamical behaviors of system (1) by doing some numerical simulations. Numerical simulations are achieved to clarify the theoretical result and display the change in the equilibrium point solution through Phase Portrait and a bifurcation diagram. Numerical simulations on the prey-predator are carried out to see the influence of fear effect on prey, addition food and predation on predator in an ecosystem. For the numerical simulations, the parameters of Table 2 are used. We have no field data for the numerical simulations. The values of the selected parameters only hypothetical data are adjusted and to clarify the dynamics to be shown of the system (1).

Table 2. The parameter value used for the simulation.

Parameter	Description	Value
r	The growth rate of prey	5.62
f	The fear effect of prey on predator	0.28
α	The predation rate of predators	1.25
m	The environmental protection for prey	1.23
n	The relative ability of the predator to detect additional food	0.83
A	The additional food for the predators	1.97
β	The conversion growth rate of the predator from the prey	1.16
μ	The natural mortality of predator	0.3

The following hypothetical set of parameter values, which are biologically reasonable, is used to numerically settle the system (1). For simulation, we take the same parameter values as in Table 2. All equilibrium points are $E_1(0,0)$, $E_2(0,3.45728)$, $E_3(5.58103,19.45626)$

7. Diagram Bifurcation

Numerical continuity is performed on an equilibrium solution with a variation in one of the three parameters discussed in this section to indicate a change in the type of stability at equilibrium. To further investigate the influence of biological factors, we perform a bifurcation analysis by numerically focusing on three parameters: the fear effect on prey parameter f, the additional food on predator parameter A, and parameter β the conversion growth rate of the predator from the prey with Beddington-DeAngelis functional response.

We investigate the change in dynamical behaviors by doing some numerical scenarios. The first numerically, We perform a numerical continuation by selecting the parameter of fear effect for the prey, which is f, while the values of the other parameters remain constant. The result of varying the parameter f on the dynamics of the system (1) is presented in Figure 1. By forward and backward continuation techniques, we find the Hopf and transcritical bifurcation. Figure 1 shows the Hopf and transcritical bifurcation of the system (1) using Matcont.

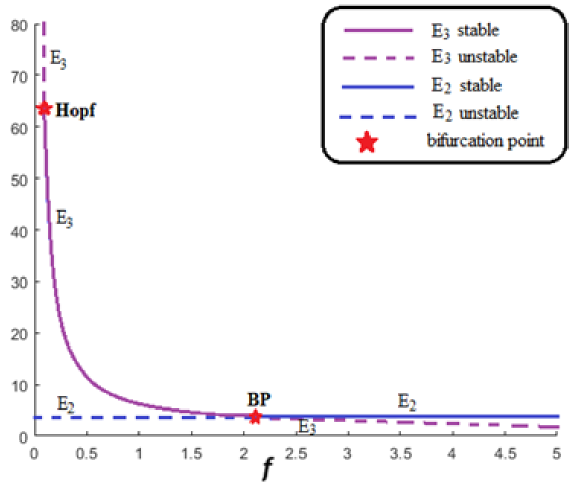


Figure 1. Bifurcation diagram driven by parameter the fear effect on the prey.

According to Figure 1, system (1) has interesting dynamic behavior in the form of a bifurcation diagram. The bifurcation diagram in Figure 1 illustrates the exchange between the interior point of the equilibrium (E_3) and the point of extinction of the prey population (E_2). Using forward continuation techniques, we find the Hopf bifurcation when $f = 0.081119$ and a stable limit cycle view near the point of interior equilibrium in Figure 1. As the parameter f increases reflecting a stronger fear effect on the prey population occurrence the other phenomenon shows Branch point (BP). This BP indicates a transcritical bifurcation when $f = 2.088891$. Changes in the system solution occur at the interior equilibrium point (E_3), which was initially stable but becomes unstable. Consequently, the point shifts to equilibrium of the extinction of the prey population (E_2), which is stable. Intensifying fear effects can cause prey to limit their exposure to predators.

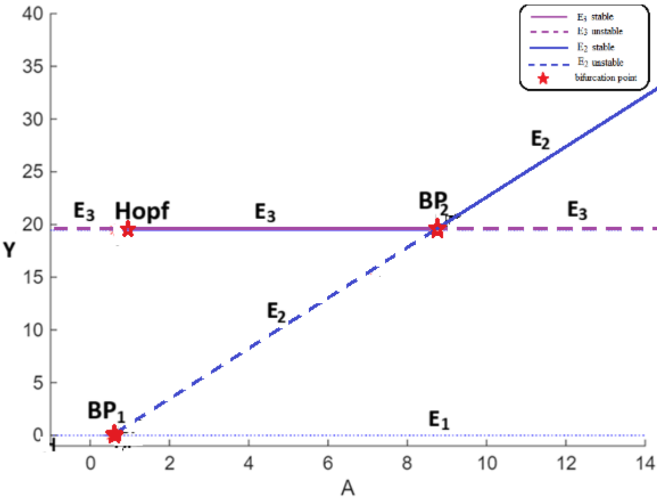


Figure 2. Bifurcation diagram driven by parameter the additional food on the predator.

To investigate the impact of addition food on predator on the dynamical behavior of system (1). As shown in Figure 2, the system (1) undergoes a Transcritical bifurcation at the critical when $A = 0.516952$ marking a significant shift in the nature and stability of its equilibrium point. As parameter A increases, has interesting dynamic behavior in the form of a bifurcation diagram. Emergent the Hopf bifurcation when $A = 0.625978$ and a stable limit cycle view near the point of interior equilibrium (E_3) in Figure 2. Using forward continuation techniques, we find the Transcritical bifurcation when $A = 8.694143$ illustrates the exchange between the interior point of the equilibrium (E_3) and the point of extinction of the prey population (E_2). The interior equilibrium loses its stability, where only predators survive. Consequently, prey populations can decline even in the absence of direct predation, while predator populations persist due to additional food.

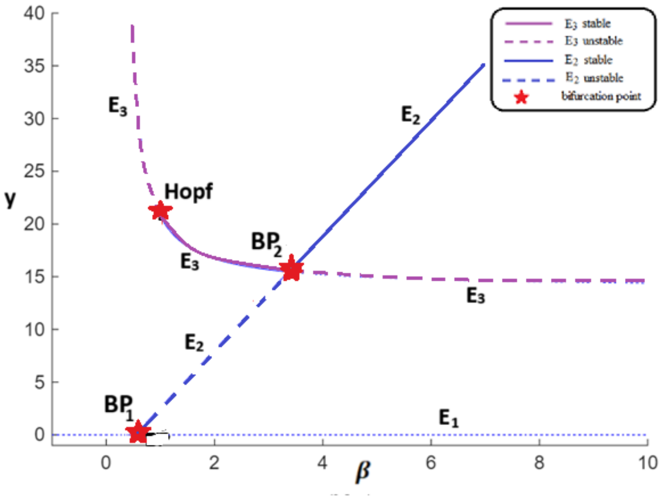


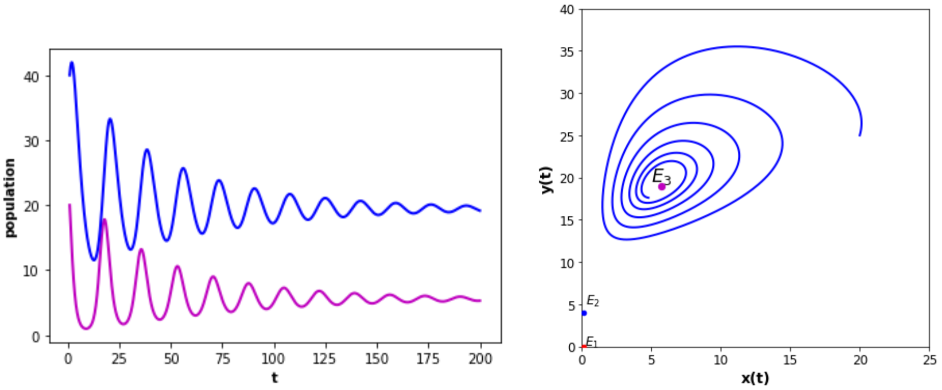
Figure 3. Bifurcation diagram driven by parameter the conversion growth rate of the predator from the prey.

As a result showing to Figure 3. The simulation using parameter β, corresponding to the conversion predation rate of the predator, the following bifurcation diagram is obtained. From Figure 3 it can be noticed that by varying the value if β, there are two bifurcations taking place. First, a Transcritical bifurcation when $\beta = 0.525674$. Therefore in the interval $0.515674 < \beta < 1.023308$ there exist three equilibrium unstable, i.e. (E_1),(E_2), and (E_3). Using forward continuation techniques, we find the Hopf bifurcation when $\beta = 1.023308$ the exchange between the interior point of the equilibrium (E_3) stable and the point of extinction of the prey population (E_2). The other phenomenon of backward continuation techniques, as β increases, Branch point (BP) appear at Figure 3. This BP indicates a transcritical bifurcation when $\beta = 3.362426$. Changes in the system solution occur at the interior equilibrium point (E_3), which was initially stable in the interval $1.023308 < \beta < 3.362426$ becomes unstable. Consequently, the point shifts to equilibrium of the extinction of the prey population (E_2), which is stable in subinterval $(\beta > 3.362426)$.

8. Phase Portrait

We obtained solutions of the results in the form of convergent series with the Runge-Kutta Orde 4 technique. The study numeric are very important to the solution of system (1) performed by Python 3.7. We perform a numerical simulation showing that the system (1) periodically undergoes the limit cycle. The phase portrait of the system (1) starting from parameter in Table 2. For the first simulation, we take the same parameter values as in Table 2. Using these parameter values, it can be shown that the equilibrium point E_1, E_3 of the system (1) exists. All equilibrium points are $E_1(0,0)$ and $E_2(0, 3.45728)$, which is unstable, $E_3(5.58103, 19.45626)$ which is stable at Figure 4b.

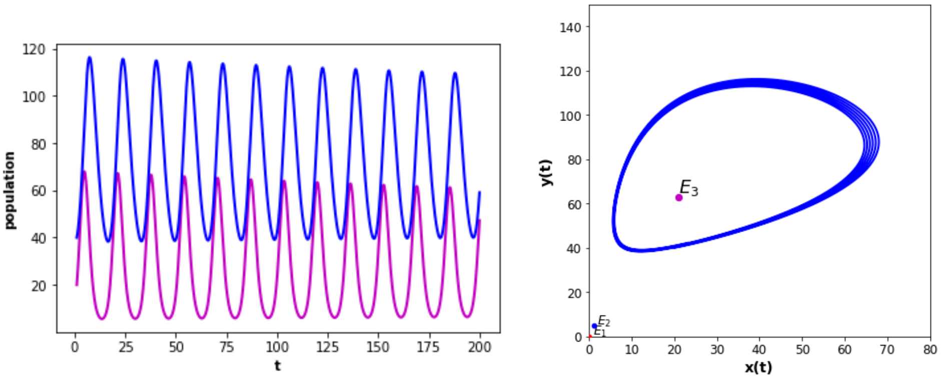
As a result, we have the phase portrait given by Figure 5 describes the predator-prey dynamic



(a) The time series of system (1) with parameter Table 2.

(b) The solution of system (1) the phase portraits with parameter in Table 2.

Figure 4. The dynamics of system (1) with parameter in Table 2.



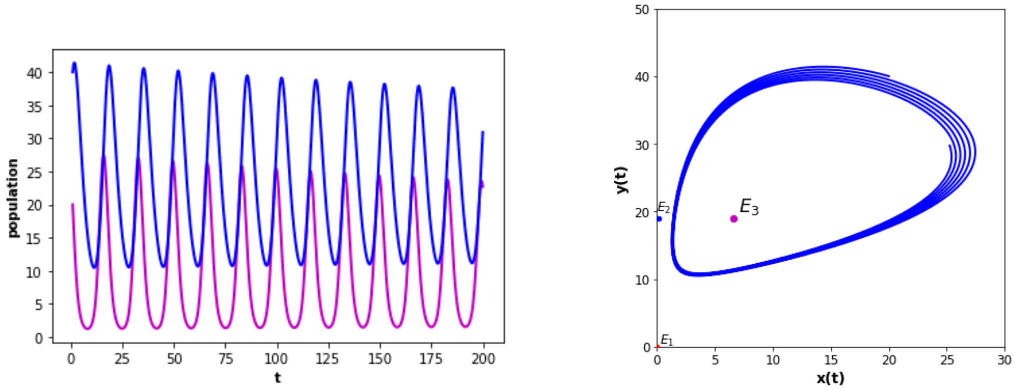
(a) The time series of system (1) indicates the limit cycle at $f = 0.081119$.

(b) The solution of system (1) the phase portraits appears Hopf Bifurcation at $f = 0.081119$.

Figure 5. The dynamics of system (1) at $f = 0.081119$.

in the system (1). The trajectories of the system (1) are shown in Figure 5 after being obtained using the parameter in Table 2 with $f = 0.081119$. We plot the time series of both population densities of the solution system (1), the prey x population is represented by the magenta line, and the predator y is represented by the blue line. The curve shows the oscillation when the amplitudes have the same size for all time series. Their population densities will eventually change periodically in nature along with time.

We also give two other simulations to show the impact of addition food on predator with A and conversion predation on predator β parameter. The phase portrait given by Figure 5 describes the predator-prey dynamic in the system (1). The trajectories of the system (1) are shown in Figure 6 after being obtained using the parameter in Table 2 with $A = 0.625978$. Figure 6b shows phase portraits that confirm that all the equilibrium points $E_1(0,0), E_2(0,19.456)$, and $E_3(6.696,19.456)$ exist with different values of A, showing that E_3 is stable. We plot the time series of both population densities of the solution system (1), the prey x population is represented by the magenta line, and the predator y is represented by the blue line. The curve shows the oscillation when the amplitudes have the same size for all time series.

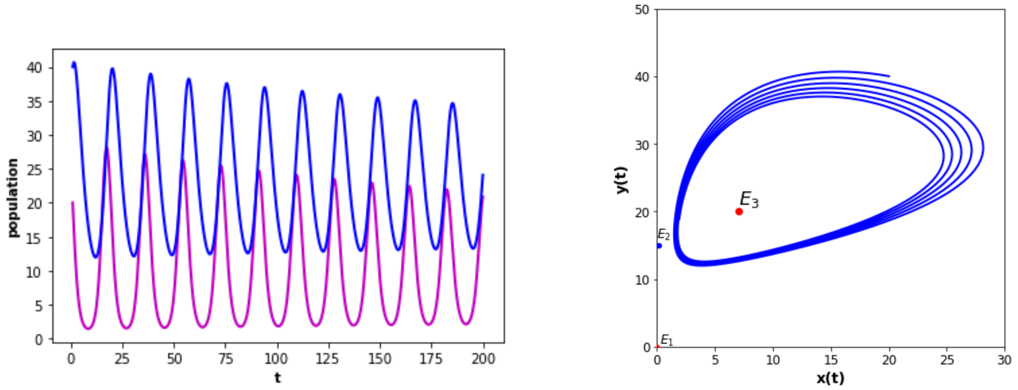


(a) The time series of system (1) indicates the limit cycle on additional food at $A = 0.625978$.

(b) The solution of system (1) the phase portraits appears Hopf Bifurcation at $A = 0.625978$.

Figure 6. The dynamics of system (1) at $A = 0.625978$.

The last simulation to show the impact of predation on predator β parameter. The phase portrait given by Figure 5 describes the predator-prey dynamic in the system (1). The trajectories of the system (1) are shown in Figure 7 after being obtained using the parameter in Table 2 with $\beta = 1.023308$. Figure 7b shows phase portraits that confirm that all the equilibrium points $E_1(0,0), E_2(0,15.461)$, and $E_3(7.381,20.508)$ exist with different values of β, showing that E_3 is stable. This ends the numerical simulations as well as all mathematical results.



(a) The time series of system (1) indicates the limit cycle on convestion predation at $\beta = 1.023308$.

(b) The solution of system (1) appears Hopf Bifurcation at $\beta = 1.023308$.

Figure 7. The dynamics of system (1) at $\beta = 1.023308$.

The stability of interior equilibrium, in which prey and predator population the presence of a stable limit cycle or periodic solution when it is plotted in a time series correspond to the dynamics of x and y population, respectively, for $0 < t < 200$. It is evident from those figures that a bounded limit cycle results from stable limit cycles. These imply that the growth dynamics of prey and predator coexistence will continue to exhibit periodic behavior for a very long time.

9. Conclusion

This study shows that dynamically considering the fear effect, the addition of food for predator and predation with Beddington-DeAngelis functional response has a significant influence on the dynamics of Detritivore as prey and predator detritivore populations. Therefore, it is imperative that ecological models combine the predation effect and additional food for predators using the Beddington–DeAngelis functional responses. The results of the analysis of the dynamics of the prey–predator described the local stability of three points of equilibrium. One equilibrium condition is always unstable, that is, the equilibrium point of extinction for all populations. The other equilibrium points are stable with certain conditions that have been proven. Finally, we see that the prey and predator population can live together under some conditions, as described in this paper. Some numerical simulation have provided not only support the analytical finding but also to show the phenomena called the Hopf bifurcation with the fear effect on prey, additional food for predators, and predation used Beddington-DeAngelis becomes the bifurcation parameter. The results of the numerical simulation also show that the model of the prey–predator interaction describes the occurrence of a Hopf bifurcation and transcritical bifurcations of the phenomena. It is shown the density for all populations can changes periodically in nature along with time.

Supplementary Information

Author Contributions. **Dian Savitri:** Conceptualization, Methodology, Formal analysis, Discussion, WritingOriginal Draft, Visualization, writing **Abadi:** Conceptualization, Formal analysis, writing-review. **Riska W Romadhonia**: Data, Discussion, software, visualization, writing-review. **Nurul Imamah Ah:** Methodology, Discussion, and Writing review. **An Nisa Salsabila:** Simulation, visualization and Writing, editing. **Donna Karnasih:** formal analysis, writing-and editing.

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Conflict of interest. The authors declare that they have no conflicts of interest to report regarding the present study.

Data availability. The data supporting the findings of this study were obtained through numerical simulations and are available from the corresponding author upon reasonable request

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